THE practice final

Problem 1. Consider the line

$$\ell(t) = (3 + t, -5 + t, 2 - 2t).$$

Parameterize a line that contains the point $(4, 0, -7)$ and meets the line $\ell(t)$ orthogonally.

Problem 2. Consider the lines:

$$\ell_1(t) = (2 + t, 5 - t)$$
$$\ell_2(t) = (2 + t, 5 + 2t)$$

(a) Parameterize two lines, one parallel to $\ell_1(t)$ and the other parallel to $\ell_2(t)$, containing the point $(1, 0)$.
(b) Use determinants to find the area of the parallelogram bounded by the four lines.

Problem 3.

(a) Find all vectors $\overline{v}$ such that $||\overline{v}|| = 5$ which form an angle of $\pi/4$ with the vector $(1, -1)$.
(b) Compute the orthogonal projections of these vectors onto $(1, -1)$.

Problem 4.

(a) Find an equation for the plane passing through the points $(1, 0, 2)$, $(3, 1, 4)$ and $(2, -1, 1)$.
(b) Find an equation for the plane parallel to the plane from part (a) that passes through the point $(4, -3, 2)$.

Problem 5. Consider the function

$$f(x, y) = x^2 + y^2 - 2(x + y) + 2.$$

Find the absolute minimum and maximum values of f in the quarter circle

$$R = \{(x, y) \in \mathbb{R}^2 | x \geq 0, y \geq 0, x^2 + y^2 \leq 2\}.$$

Problem 6. Consider the function $f(x, y)$ given by

$$f(x, y) = x^2 - y^2,$$

and the path $\overline{c}(t)$ given by

$$\overline{c}(t) = (2\cos(t), \sin(t)), \text{ for } 0 \leq t \leq 2\pi.$$

(a) Describe the geometry of the path $\overline{c}(t)$.
(b) Find the absolute max and min value of f along the path $\overline{c}(t)$.

Problem 7. Consider the function

$$f(x, y) = 2x^3 + 3x - y^2 - 1.$$

(a) Find the directional derivative of f in the direction of the vector $(1, -1)$ at the point $(-3, 1)$.
(b) Find all directions along which f neither increases nor decreases at the point $(1, 1)$. Express your answer as a list of vectors.

Problem 8. Consider the function

$$f(x, y) = 4 - x^2 - y^2.$$

(a) Sketch a level curve diagram for f in the (x, y)-plane. (Include at least 4 level curves.)
(b) Use your answer in part (a) to sketch the graph of f in $\mathbb{R}^3$.

Problem 9. Find the equation of the tangent plane to the graph of the function $f(x, y) = e^x \sin(2x - y)$ at the point $(\pi, -\pi/2)$.

Problem 10. Let $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ and $G\colon \mathbb{R}^2 \to \mathbb{R}^2$ be given by the formulas

$$F(x, y) = (3x + 2y, x^2 - 1)$$
$$G(x, y) = (x^2 - y^2, 2y^3 + 3).$$

Use the chain rule to find the derivative matrix $D(F \circ G)$ at the point $(1, 1)$.

Problem 11.

(a) Let S be the ellipsoid defined by the equation

$$x^2 + 3y^2 + z^2 = 1.$$

What is the maximum value of $f(x, y, z) = x - y + z$ on S?

(b) Let S be the ellipse defined by the equation

$$x^2 + 2y^2 = 1.$$

Let $f(x, y) = (2x - 3y)^2$. Find the maximum and minimum values of $f(x, y)$ on S.

(c) Find the global maximum and minimum values of $h(x, y) = 2x^2 + 2y^2 + 8x + 16$ on the closed disc $D = \{(x, y)|x^2 + y^2 \leq 25\}$.

Problem 12. Find the arclength of the path

$$\bar{c}(t) = \left(\frac{1}{2}t^2, \ln(\sqrt{t}), t\right)$$

for $1 \leq t \leq 2$.

Problem 13. Compute the iterated integral

$$\iint\limits_B x^2 dxdy$$

over the rectangular region $B = [0, 2] \times [0, 1]$.

Problem 14. Let T be the (filled-in) triangle in the plane with vertices $(0, 0)$, $(0, 4)$, and $(2, 2)$. Evaluate the integral:

$$\iint_T e^{x+y} dydx.$$

Problem 15. Evaluate the iterated integral

$$\int\limits_0^2 \int\limits_{y/2}^1 ye^{x^3} dxdy.$$